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On the renormalization of operator products: the scalar gluonic case

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Abstract: In this paper we study the renormalization of the product of two operators $O_1 = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu}$ in QCD. An insertion of two such operators $O_1(x)O_1(0)$ into a Greens function produces divergent contact terms for $x \to 0$.

In the course of the computation of the operator product expansion (OPE) of the correlator of two such operators $i\int \mathrm{d}^4x\, e^{iqx} T\{\, O_1(x)O_1(0)\}$ to three-loop order [1, 2] we discovered that divergent contact terms remain not only in the leading Wilson coefficient C_0, which is just the VEV of the correlator, but also in the Wilson coefficient C_1 in front of O_1. As this correlator plays an important role for example in QCD sum rules a full understanding of its renormalization is desireable.

This work explains how the divergences encountered in higher orders of an OPE of this correlator should be absorbed in counterterms and derives an additive renormalization constant for C_1 from first principles and to all orders in perturnbation theory. The method to derive the renormalization of this operator product is an extension of the ideas of [3] and can be generalized to other cases.

Keywords: QCD, Sum Rules, Renormalization

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Contents

1 Introduction: The scalar gluonic operator O_1 and its correlator

Local operators, i.e. products of fields at the same point in space-time, play an important role in quantum field theory (QFT) as they serve as building blocks for Lagrangians and Greens functions. The bilocal correlator of two such local operators is an important object in applications of QFT, such as sum rules. In this paper we study the renormalization of the scalar gluonic operator

$$O_1(x) := -\frac{1}{4}G^{a\,\mu\nu}G^a_{\mu\nu}(x) \tag{1.1}$$

constructed from the field strength tensor of QCD

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu. \tag{1.2}$$

The operator (1.1) appears in the massless QCD Lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} - \frac{1}{2\lambda}(\partial_\mu A^{a\,\mu})^2 + \partial_\rho \bar{c}^a \partial^\rho c^a + g_s f^{abc}\partial_\rho \bar{c}^a A^b c^c \\ &+ \bar{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{\partial}} - m\right)\psi + g_s \bar{\psi}\not{A}^a T^a \psi\end{aligned} \tag{1.3}$$

where T^a are the generators and f^{abc} the structure constants of the gauge group.

A renormalized version of this operator, i.e. one which gives finite results if inserted into a Greens function, was obtained in [3, 4]. If we only consider matrix elements with physical external states and $m = 0$ it can be renormalized multiplicatively:

$$[O_1] = Z_{11}O_1^{\mathrm{B}} = -\frac{Z_{11}}{4}G^{\mathrm{B}\,a\,\mu\nu}G^{\mathrm{B}\,a}_{\mu\nu} \tag{1.4}$$

where $[\dots]$ marks the renormalized operator and the index B bare quantities, which means that all fields and couplings are replaced by bare ones. The renormalization constant derived in [3, 4]

$$Z_{11} = 1 + \alpha_s \frac{\partial}{\partial \alpha_s} \ln Z_{\alpha_s} = \left(1 - \frac{\beta(\alpha_s)}{\varepsilon}\right)^{-1} \tag{1.5}$$

can be expressed through the beta function

$$\beta(\alpha_s) = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \ln \alpha_s = -\sum_{i\geq 0} \beta_i \left(\frac{\alpha_s}{\pi}\right)^{i+1} \tag{1.6}$$

and at first order in α_s is equal to the renormalization constant for α_s: $Z_{11} = Z_{\alpha_s} + \mathcal{O}(\alpha_s^2)$.

The bilocal correlator of this operator is defined as

$$\widehat{\Pi}^{\mathrm{GG}}(q^2) := i \int \mathrm{d}^4x \, e^{iqx} T\{\,[O_1](x)[O_1](0)\} = i\, Z_{11}^2 \int \mathrm{d}^4x \, e^{iqx} T\{\, O_1^{\mathrm{B}}(x) O_1^{\mathrm{B}}(0)\}, \tag{1.7}$$

The OPE of the correlator (1.7) (considering only scalar operators) reads

$$\begin{aligned} \widehat{\Pi}^{\mathrm{GG}}(q^2) &= q^4\, C_0^{\mathrm{GG}}(q^2)\, 1 \;+\; C_1^{\mathrm{GG\,B}}(q^2)\, O_1^{\mathrm{B}} \;+\; \sum_i C_i^{\mathrm{GG\,B}}(q^2) O_i^{\mathrm{B}} \;+\; \mathcal{O}\left(\frac{1}{q^2}\right) \\ &= q^4\, C_0^{\mathrm{GG}}(q^2)\, 1 \;+\; C_1^{\mathrm{GG}}(q^2)\, [O_1] \;+\; \sum_i C_i^{\mathrm{GG}}(q^2) [O_i] \;+\; \mathcal{O}\left(\frac{1}{q^2}\right), \end{aligned} \tag{1.8}$$

where the sum goes over a set of mass dimension four operators which form a suitable basis together with O_1^{B} or $[O_1]$ (see section 2). For sum rules (see e.g. [5]) we are usually interested in the vacuum expectation value (VEV) of the correlator

$$q^4\, \Pi^{\mathrm{GG}}(q^2) = \langle 0 | \widehat{\Pi}^{\mathrm{GG}}(q^2) | 0 \rangle \tag{1.9}$$

for large Euclidean momenta $-q^2 \gg 0$. As the VEV of unphysical operators[1] vanishes we can restrict ourselves to physical operators.

C_0^{GG} is known at four-loop level from [6] and C_1^{GG} at three-loop level from [2]. In [1, 2] it was discovered, however, that the described renormalization procedure does not yield a finite result for C_1^{GG} starting from two-loop level. These divergent terms are proportional to $\delta^{(4)}(x)$ in x-space and hence stem from the point where both operators O_1 in the correlator (1.7) are at the same point $x = 0$. For this reason they are called contact terms.[2]

The complete renormalization of the operator product of two O_1 is also desirable for phenomenological applications in effective theories. An important example is double Higgs production in the framework of an effective theory with $m_t \to \infty$. Having integrated out the top loops the resulting vertices are $\propto O_1 H$ and $\propto O_1 HH$, where H is the Higgs field (see e.g. [8]). the new renormalization constant Z_{11}^{L} defined below in (3.4) will be needed in

[1] These are gauge dependent operators or operators which vanish due to equations of motion.

[2] In the case of the correlator of the pseudoscalar operator $\tilde{O}_1(x) := G^{a\,\mu\nu} G^{a\,\rho\sigma} \varepsilon_{\mu\nu\rho\sigma}$ it was proven in [7] that no contact terms can apear in C_1 which was explicitly confirmed in a three-loop calculation of this quantitiy in the same paper.

a counterterm $\propto O_1 HH$ if two effective vertices $\propto O_1 H$ are inserted into two-loop diagrams with two external gluons, i.e. starting at one loop-order higher than the results presented in [8].

The paper is structured as follows: In section 2 the renormalization of Greens functions with one insertion of O_1 is reviewed following the ideas of [3]. This method is then extended in section 3 in order to renormalize the product of two such operators followed by the application of the found result to the OPE (1.8) in section 3 explaining the contact term in $C_1^{\rm GG}$ [1, 2]. We finish with some conclusions and acknowledgments.

2 Renormalization of O_1

For the purpose of this and the next section we rescale the field $A_\mu^a \to \frac{A_\mu^a}{g}$ transforming (1.3) into

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4g_s^2}\,G_{\mu\nu}^a G^{a\,\mu\nu} - \frac{1}{2\lambda g_s^2}\,(\partial_\mu A^{a\,\mu})^2 + \partial_\rho \bar{c}^a \partial^\rho c^a + f^{abc}\partial_\rho \bar{c}^a A^b c^c \\ &+ \bar{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{\partial}} - m\right)\psi + \bar{\psi}\not{A}^a T^a \psi\end{aligned} \tag{2.1}$$

with the rescaled field strength tensor

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c. \tag{2.2}$$

We define the renormalization prescriptions

$$A_\mu^{\mathrm{B}\,a} = A_\mu^a \frac{Z_1}{Z_3}, \quad c^{\mathrm{B}\,a} = c^a\sqrt{\tilde{Z}_3}, \quad \psi^{\mathrm{B}} = \psi\sqrt{Z_2}, \quad m^{\mathrm{B}} = m Z_m, \quad g_s^{\mathrm{B}} = g_s \frac{Z_1}{Z_3^{3/2}} \equiv g_s Z_g, \tag{2.3}$$

where $Z_g^2 = Z_{\alpha_s}$ with $\alpha_s = \frac{g_s^2}{4\pi}$. Hence

$$\frac{A_\mu^{\mathrm{B}\,a}}{g_s^{\mathrm{B}}} = \frac{A_\mu^a}{g_s}\sqrt{Z_3} \tag{2.4}$$

which is just the renormalization procedure for A_μ^a in the original Lagrangian (1.3). Z_1, Z_2, Z_3, $\tilde{Z}_3$ and Z_m are therefore the usual renormalization constants of QCD.[3] The bare Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\mathrm{B}} = &-\frac{1}{4(g_s^{\mathrm{B}})^2}\,G_{\mu\nu}^{\mathrm{B}\,a} G^{\mathrm{B}\,a\,\mu\nu} - \frac{1}{2\lambda\, Z_3\,(g_s^{\mathrm{B}})^2}\,(\partial_\mu A^{\mathrm{B}\,a\,\mu})^2 + \partial_\rho \bar{c}^{\mathrm{B}\,a}\partial^\rho c^{\mathrm{B}\,a} \\ &+ f^{abc}\partial_\rho \bar{c}^{\mathrm{B}\,a} A^{\mathrm{B}\,b} c^{\mathrm{B}\,c} + \bar{\psi}^{\mathrm{B}}\left(\frac{i}{2}\overleftrightarrow{\not{\partial}} - m_{\mathrm{B}}\right)\psi^{\mathrm{B}} + \bar{\psi}^{\mathrm{B}}\not{A}^{a\,\mathrm{B}} T^a \psi^{\mathrm{B}}.\end{aligned} \tag{2.5}$$

Finite results for Greens functions are usually obtained by applying the R-Operation (see e. g. [9, 10]) to the unrenormalized Greens function or equivalently by using the bare

[3]The same as with unrescaled A_μ^a and the definition $A_\mu^{\mathrm{B}\,a} = A_\mu^a\sqrt{Z_3}$.

Lagrangian in which a counterterm for every operator in the Lagrangian is defined. Finite Greens functions are derived from the generating functional of the path integral formalism

$$Z^{\mathrm{R}} = \mathbf{R} \int \mathrm{d}\Phi \, \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}+J\cdot\Phi)} \tag{2.6}$$
$$= \int \mathrm{d}\Phi \, \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \tag{2.7}$$

with the multiplets of all fields in the Lagrangian and the respective external currents

$$\Phi := (\frac{A^\mu}{g_s}, \bar{c}, c, \bar{\psi}, \psi), \quad J := (J_\mu, J_{\bar{c}}, J_c, J_{\bar{\psi}}, J_\psi) \tag{2.8}$$

and the integration measure

$$\mathrm{d}\Phi := \mathrm{d}A^\mu \, \mathrm{d}\bar{c} \, \mathrm{d}c \, \mathrm{d}\bar{\psi} \, \mathrm{d}\psi. \tag{2.9}$$

A finite Greens function with the insertion of a local operator $\widetilde{O}_i(q) = \int \mathrm{d}^4x \, e^{iqx} O_i(x)$ is obtained as

$$Z^R_{O_i}(q) = \mathbf{R} \int \mathrm{d}\Phi \, \widetilde{O}_i(q) \, e^{i\int \mathrm{d}^4x(\mathcal{L}+J\Phi)} \tag{2.10}$$

which can also be written in terms of the bare Lagrangian and a superposition of bare local operators

$$Z^R_{O_i} = Z_{ij} \int \mathrm{d}\Phi \, \widetilde{O}^B_j(q) \, e^{i\int \mathrm{d}^4x(\mathcal{L}_B+J\Phi)} \tag{2.11}$$

In MS-like schemes the renormalization constants for these operators do not depend on q and hence we set $q = 0$. The important point now is that in renormalization schemes based on minimal subtraction the R-Operation commutes with the operation of taking derivatives wrt the parameters of the theory $g_s, \lambda, \ldots$ and wrt external currents. An operator insertion of $\widetilde{O}_1 \equiv \widetilde{O}_1(0)$ in a Greens function can be obtained [3] by applying the operation

$$\mathbf{D}_1 := \frac{1}{i}\left(\lambda\frac{\partial}{\partial\lambda} - \frac{1}{2}g_s\frac{\partial}{\partial g_s} - \frac{1}{2}J_\mu\frac{\delta}{\delta J_\mu}\right) \tag{2.12}$$

to (2.6):

$$\mathbf{D}_1 \, Z^{\mathrm{R}} = \mathbf{D}_1 \, \mathbf{R} \int \mathrm{d}\Phi \, \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}+J\cdot\Phi)} = \mathbf{R} \, \mathbf{D}_1 \int \mathrm{d}\Phi \, \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}+J\cdot\Phi)} \tag{2.13}$$
$$= \mathbf{R} \int \mathrm{d}\Phi \, \widetilde{O}_1 \, \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}+J\cdot\Phi)}$$

Using the representation (2.7) of Z_{R} we find that this equals

$$\mathbf{D}_1 \, Z^{\mathrm{R}} = \int \mathrm{d}\Phi \left\{\left(\lambda\frac{\partial}{\partial\lambda} - \frac{1}{2}g_s\frac{\partial}{\partial g_s}\right)\mathcal{L}_{\mathrm{B}}\right\} \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \tag{2.14}$$
$$= \int \mathrm{d}\Phi \left\{\sum_{i=1}^{5} Z_{1i}\widetilde{O}^{\mathrm{B}}_i\right\} \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \tag{2.15}$$

A suitable basis of mass dimension four operators was given in [3]:

$$
\begin{aligned}
O_1 &= -\frac{1}{4g_s^2} G^{a\,\mu\nu} G^a_{\mu\nu}, &(2.16)\\
O_2 &= m\bar{\psi}\psi, &(2.17)\\
O_3 &= \bar{\psi}\left(\frac{i}{2}\overleftrightarrow{\not{\partial}} - m\right)\psi, &(2.18)\\
O_4 &= A^a_\nu \left\{\left(\delta^{ab}\partial_\mu - g f^{abc} A^c_\mu\right) G^{b\,\mu\nu} + \bar{\psi}T^a\gamma^\nu\psi\right\} &(2.19)\\
&\quad - \left(\partial_\mu \bar{c^a}\right)\left(\partial^\mu c^a\right), \\
O_5 &= \left\{\left(\delta^{ab}\partial_\mu - f^{abc} A^c_\mu\right)\partial^\mu \bar{c^a}\right\} c^b. &(2.20)
\end{aligned}
$$

Using (2.14) and collecting the coefficients in (2.15) we find the renormalization constants Z_{1i}. The second line of (2.13) gives us the renormalized operator $[O_1]$, such that from (2.13) and (2.15) we have $[O_1] = Z_{1j} O_j^{\mathrm{B}}$. Similarly the renormalization constants for the other operators

$$[O_i] = Z_{ij} O_j^{\mathrm{B}}. \tag{2.21}$$

are derived:

$$Z_{ij} = \delta_{ij} + \overline{D}_i \ln \overline{Z}_j \qquad (i,j \in \{1,\dots,5\}) \tag{2.22}$$

with

$$
\begin{aligned}
\overline{D}_1 &= \lambda\frac{\partial}{\partial\lambda} - \alpha_s\frac{\partial}{\partial\alpha_s},\ \ \overline{D}_2 = -m\frac{\partial}{\partial m},\ \ \overline{D}_3 = 0,\ \ \overline{D}_4 = 2\lambda\frac{\partial}{\partial\lambda},\ \ \overline{D}_5 = 0, &(2.23)\\
\overline{Z}_1 &= Z_\alpha^{-1},\ \ \overline{Z}_2 = Z_m^{-1},\ \ \overline{Z}_3 = Z_2,\ \ \overline{Z}_4 = Z_1 Z_3^{-1},\ \ \overline{Z}_5 = Z_3 Z_1^{-1}. &(2.24)
\end{aligned}
$$

These were first found in [3] and rederived for this study.[4] The gauge-invariant operators (2.16) and (2.17) are physical operators of class I according to the classification from [3, 11]. In physical matrix elements the class I operators do not vanish whereas the gauge-invariant class II^a operator (2.18) vanishes due to an equation of motion. The non gauge-invariant operators (2.19) and (2.20) are of class II^b and vanish due to a BRST identity in physical matrix elements. Hence in the massless case O_1 is renormalized multiplicatively with Z_{11} as given in (1.5). In the following we set $m = 0$.

3 Renormalization of the product of two operators O_1

We now want to apply this procedure in order to derive the renormalization constants for the insertion of two operators $\widetilde{O}_1$ into a Greens function. First (using (2.13)) we notice that

$$\mathbf{D}_1\,\mathbf{D}_1\,Z^{\mathrm{R}} + i\,\mathbf{D}_1\,Z^{\mathrm{R}} = \mathbf{R}\int \mathrm{d}\Phi\,\widetilde{O}_1\,\widetilde{O}_1\,\mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}+J\cdot\Phi)}. \tag{3.1}$$

[4] For the coefficients of the unphysical operators an additional "counting identity" is needed for which we refer to [3].

On the other hand

$$
\begin{aligned}
(\mathbf{D}_1\,\mathbf{D}_1 + i\mathbf{D}_1)\; Z^{\mathrm{R}} &= (\mathbf{D}_1\,\mathbf{D}_1 + i\,\mathbf{D}_1)\;\int \mathrm{d}\Phi\, \mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \\
&= (\mathbf{D}_1 + i)\;\int \mathrm{d}\Phi\,\left\{\sum_{i=1}^{5} Z_{1i}\widetilde{O}_i^{\mathrm{B}}\right\}\,\mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \\
&= \sum_{i,j=1}^{5} Z_{1i}Z_{1j}\int \mathrm{d}\Phi\,\widetilde{O}_i^{\mathrm{B}}\widetilde{O}_j^{\mathrm{B}}\,\mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \\
&+ \sum_{i=1}^{5}((\mathbf{D}_1 Z_{1i}) + iZ_{1i})\;\int \mathrm{d}\Phi\,\widetilde{O}_i^{\mathrm{B}}\,\mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)} \\
&+ \sum_{i=1}^{5} Z_{1i}\;\int \mathrm{d}\Phi\,\left(\mathbf{D}_1\widetilde{O}_i^{\mathrm{B}}\right)\,\mathrm{e}^{i\int \mathrm{d}^4x\,(\mathcal{L}_{\mathrm{B}}+J\cdot\Phi)}.
\end{aligned}
\tag{3.2}
$$

This means that appart from the expected term $\sum\limits_{i,j=1}^{5} Z_{1i}Z_{1j}O_i^{\mathrm{B}}O_j^{\mathrm{B}} = [O_1][O_1]$ linear (L) terms of the form $i\sum\limits_i Z_{1i}^{\mathrm{L}}O_i^{\mathrm{B}}$ with new renormalization constants Z_{1i}^{L} will in general contribute to the renormalization of an operator product:

$$
[O_1(x)O_1(0)] = [O_1(x)][O_1(0)] + i\delta(x)\sum_i Z_{1i}^{\mathrm{L}}O_i^{\mathrm{B}}(0). \tag{3.3}
$$

A renormalized correlator should hence be defined as

$$
i\int \mathrm{d}^4x\, e^{iqx}T\{\,[O_1(x)O_1(0)]\} = i\int \mathrm{d}^4x\, e^{iqx}T\{\,[O_1(x)][O_1(0)]\} - \sum_i Z_{1i}^{\mathrm{L}}O_i^{\mathrm{B}}. \tag{3.4}
$$

We can again compute the first or second line of (3.2) and collect all fields and renormalization constants into local operators. In order to simplify the calculation we note that

$$
\int \mathrm{d}^4x\,\mathcal{L}_{\mathrm{B}} = \int \mathrm{d}^4x\left(O_1^{\mathrm{B}}(x) + O_3^{\mathrm{B}}(x) - O_5^{\mathrm{B}}(x) - \frac{1}{2\lambda g_s^2}\left(\partial_\mu A^{a\,\mu}(x)\right)^2\right). \tag{3.5}
$$

From (2.14) and (2.15) we find

$$
\mathbf{D}_1\left(\widetilde{O}_1^{\mathrm{B}} + \widetilde{O}_3^{\mathrm{B}} - \widetilde{O}_5^{\mathrm{B}}\right) = \sum_i Z_{1i}\widetilde{O}_i \tag{3.6}
$$

We solve this for $\mathbf{D}_1\left(\widetilde{O}_1^{\mathrm{B}}\right)$ and plug it into the last three lines of (3.2). Then we discard all unphysical operators as well as their derivatives wrt to g_s and λ as these will not contribute to physical matrix elements.[5] This yields the result

$$
\begin{aligned}
Z_{11}^{\mathrm{L}} &= \frac{g_s^2}{2}\frac{Z_g''(g_s)}{Z_g} - \frac{3g_s^2}{2}\left(\frac{Z_g'(g_s)}{Z_g}\right)^2 - \frac{g_s}{2}\frac{Z_g'(g_s)}{Z_g} &(3.7)\\
&= -2\alpha_s^2\left(\frac{Z_{\alpha_s}'(\alpha_s)}{Z_{\alpha_s}}\right)^2 + \alpha_s^2\frac{Z_{\alpha_s}''(\alpha_s)}{Z_{\alpha_s}}. &(3.8)
\end{aligned}
$$

[5]For a full set of renormalization constants including the unphysical ones it is necessary to extend the set of unphysical operators as not all derivatives of O_3^{B}, O_4^{B}, O_5^{B} wrt g_s and λ can be reabsorbed in exactly these operators.

We showed that the idea of [3] for the derivation of renormalization constants for dimension four operators can also be used for the derivation of renormalization constants of two such operator insertions. This method can be used for any operator as long as a combination of derivatives wrt to external currents and parameters of the theory exists which produces an insertion of this operator into a Greens function starting from the generating functional Z_R of the theory. In general, care has to be taken that contributions to different physical and unphysical operators are separated. Here we considered only one physical operator and discarded the unphysical ones.

Note that the procedure of inserting zero-momentum operators into Z_R will produce only renormalization constants which are momentum independent and do not vanish for $q \to 0$. Hence we do not find a counterterm here which absorbs the contact terms in $C_0^{\rm GG}$ of (1.8). Such a counterterm Z_0 is $\propto q^4$ in momentum space. Accounting for this we can complete (3.4) and (3.3):

$$i \int \mathrm{d}^4x\, e^{iqx} T\{\, [O_1(x)O_1(0)]_{\rm full}\} = i \int \mathrm{d}^4x\, e^{iqx} T\{\, [O_1(x)][O_1(0)]\} - \sum_i Z_{1i}^{\rm L} O_i^{\rm B} - q^4 Z_0, \quad (3.9)$$

$$[O_1(x)O_1(0)] = [O_1(x)][O_1(0)] + i\delta(x) Z_{1i}^{\rm L} O_i^{\rm B}(0) + i\Box_x^2 \delta(x) Z_0. \quad (3.10)$$

But deriving Z_0 from first principles is not within the reach this method.

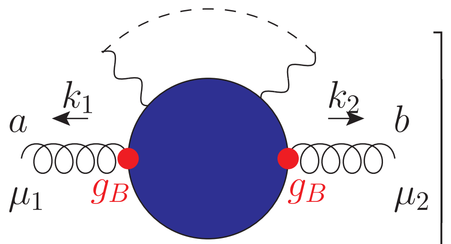
4 Application to the OPE of the O_1O_1−correlator

In [1, 2] the Wilson coefficient $C_1^{\rm GG}(q^2)$ was computed using the method of projectors [12, 13]. A projector $\mathbf{P}$ is applied to both sides of (1.8) which has the property $\mathbf{P}\{O_1^{\rm B}\} = 1$ and $\mathbf{P}\{O_i^{\rm B}\} = 0$ for $i \neq 1$. Thus we computed the bare Wilson coefficient via

$$\mathbf{P}\{i \int \mathrm{d}^4x\, e^{iqx} T\{\, [O_1](x)[O_1](0)\}\} = \sum_i C_i^{\rm B}(q)\, \mathbf{P}\{O_i^{\rm B}\} \equiv C_1^{\rm B}(q). \quad (4.1)$$

with the projector $\mathbf{P}$ defined as:[6]

$$C_1^{\rm B}(q) = \frac{\delta^{ab}}{n_g} \frac{g^{\mu_1\mu_2}}{(D-1)} \frac{1}{D} \frac{\partial}{\partial k_1} \cdot \frac{\partial}{\partial k_2} \left[\quad \right] \Bigg|_{k_i=0}, \quad (4.2)$$



where the blue circle represents the the sum of all bare Feynman diagrams which become 1PI after formal gluing (depicted as a dotted line in (4.2)) of the two external lines representing the operators on the lhs of the OPE. These external legs carry the large Euclidean momentum q. If we use the fully renormalized current (3.4) we find

$$\mathbf{P}\{i \int \mathrm{d}^4x\, e^{iqx} T\{\, [O_1(x)O_1](0)\}\} = \sum_i C_i^{\rm B}(q)\, \mathbf{P}\{O_i^{\rm B}\} - \sum_i Z_{1i}^{\rm L} \mathbf{P}\{O_i^{\rm B}\} \equiv C_1^{\rm B}(q) - Z_{11}^{\rm L} \quad (4.3)$$

[6]The Feynman diagram was drawn with the Latex package Axodraw [14].

and using (1.4) we find a fully renormalized Wilson coefficient as

$$C_1^{\text{ren}} = \frac{1}{Z_{11}} C_1^{\text{B}}(q) - \frac{Z_{11}^{\text{L}}}{Z_{11}} \tag{4.4}$$

From (1.5) and (3.7) we compute[7]

$$\begin{aligned}\frac{Z_{11}^{\text{L}}}{Z_{11}} = {} & \frac{a_s^2}{\varepsilon}\left[-\frac{17C_A^2}{24} + \frac{5C_A n_f T_F}{12} + \frac{C_F n_f T_F}{4}\right] \\ & + \frac{a_s^3}{\varepsilon}\left[\frac{1415C_A^2 n_f T_F}{864} - \frac{2857C_A^3}{1728} + \frac{205C_A C_F n_f T_F}{288} - \frac{79C_A n_f^2 T_F^2}{432} - \frac{C_F^2 n_f T_F}{16} - \frac{11C_F n_f^2 T_F^2}{72}\right] \\ & + \frac{a_s^3}{\varepsilon^2}\left[-\frac{89C_A^2 n_f T_F}{144} + \frac{187C_A^3}{288} - \frac{11C_A C_F n_f T_F}{48} + \frac{5C_A n_f^2 T_F^2}{36} + \frac{C_F n_f^2 T_F^2}{12}\right]\end{aligned} \tag{4.5}$$

which is exactly the contact term observed in [1, 2]. Using

$$\alpha_s \frac{Z'_{\alpha_s}(\alpha_s)}{Z_{\alpha_s}} = -\frac{\beta(\alpha_s)}{\beta(\alpha_s) - \varepsilon} \tag{4.6}$$

we arrive at

$$\frac{Z_{11}^{\text{L}}}{Z_{11}} = -a_s^2 \frac{\beta_1}{\varepsilon} + a_s^3\left(\frac{\beta_0\beta_1}{\varepsilon^2} - \frac{2\beta_2}{\varepsilon}\right) + a_s^4\left(-\frac{\beta_0^2\beta_1}{\varepsilon^3} + \frac{\beta_1^2 + 2\beta_0\beta_2}{\varepsilon^2} - \frac{3\beta_3}{\varepsilon}\right) + \mathcal{O}(\alpha_s^5) \tag{4.7}$$

and as already suspected in [2] the contact term $\frac{Z_{11}^{\text{L}}}{Z_{11}}$ or Z_{11}^{L} can indeed be expressed through the QCD β-function to all orders, namely

$$Z_{11}^{\text{L}} = \frac{1}{\varepsilon}\frac{-\beta(\alpha_s) + \alpha_s\beta'(\alpha_s)}{\left(1 - \frac{\beta(\alpha_s)}{\varepsilon}\right)^2} = \frac{1}{\varepsilon}\left(1 - \frac{\beta(\alpha_s)}{\varepsilon}\right)^{-2} \alpha_s^2 \frac{\partial}{\partial\alpha_s}\left[\frac{\beta(\alpha_s)}{\alpha_s}\right], \tag{4.8}$$

$$\frac{Z_{11}^{\text{L}}}{Z_{11}} = \frac{1}{\varepsilon}\frac{-\beta(\alpha_s) + \alpha_s\beta'(\alpha_s)}{1 - \frac{\beta(\alpha_s)}{\varepsilon}} = \frac{1}{\varepsilon}\left(1 - \frac{\beta(\alpha_s)}{\varepsilon}\right)^{-1} \alpha_s^2 \frac{\partial}{\partial\alpha_s}\left[\frac{\beta(\alpha_s)}{\alpha_s}\right]. \tag{4.9}$$

Following the prescription of [17] we can derive the anomalous dimensions of the Wilson coefficients of the correlator (3.9) written as[8]

$$\widehat{\Pi}_{\text{full}}^{\text{GG}} := iZ_{11}^2 \int \mathrm{d}^D x\, e^{iqx} T\{\, O_1^{\text{B}}(x) O_1^{\text{B}}(0)\} - Z_{11}^{\text{L}} O_1^{\text{B}} - (\mu)^{-2} q^4 Z_0, \tag{4.10}$$

We find

$$\mu^2 \frac{d}{d\mu^2}\widehat{\Pi}_{\text{full}}^{\text{GG}} = 2\gamma_{11}\, \mu^2 \frac{d}{d\mu^2}\widehat{\Pi}_{\text{full}}^{\text{GG}} + \gamma_{11}^L\,[O_1] + \gamma_0\, q^4\left(\mu^{-2\varepsilon}\right) \tag{4.11}$$

[7] For this we need the QCD β-function at three-loop order [15, 16]. All given results are in the $\overline{\text{MS}}$-scheme. We define $a_s = \frac{\alpha_s}{\pi} = \frac{g_s^2}{4\pi^2}$ and $l_{\mu q} = \ln\left(\frac{\mu^2}{-q^2}\right)$, where μ is the $\overline{\text{MS}}$ renormalization scale. The number of active quark flavours is denoted by n_f, C_F and C_A are the quadratic Casimir operators of the quark and the adjoint representation of the gauge group and n_g is the dimension of the adjoint representation. T_F is defined through the relation $\mathbf{Tr}\left(T^a T^b\right) = T_F \delta^{ab}$.

[8] The renormalization scale μ, which was omitted before for convenience, is carefully reintroduced. $D = 4 - 2\varepsilon$ is the space-time dimension and $O_1^{\text{B}}(x)$ has mass dimension D.

and the anomalous dimensions are found to be

$$\gamma_{11} = \mu^2 \frac{d \log Z_{11}}{d\mu^2}, \tag{4.12}$$

$$\gamma_{11}^L = \left(-\mu^2 \frac{dZ_{11}^L}{d\mu^2} + 2\gamma_{11} Z_{11}^L \right) \frac{1}{Z_{11}}, \tag{4.13}$$

$$\gamma_0 = -\mu^2 \frac{dZ_0}{d\mu^2} + (2\gamma_{11} + \varepsilon) Z_0. \tag{4.14}$$

Applying these equations and the the well-known relation

$$\mu^2 \frac{d}{d\mu^2} = \alpha_s \left(\beta(\alpha_s) - \varepsilon \right) \frac{\partial}{\partial \alpha_s} + \mu^2 \frac{\partial}{\partial \mu^2} \tag{4.15}$$

to (4.8) and (1.5) we find (in the limit $\varepsilon \to 0$):

$$\gamma_{11} = -\alpha_s \frac{\partial}{\partial \alpha_s} \beta(\alpha_s), \tag{4.16}$$

$$\gamma_{11}^L = \alpha_s^2 \frac{\partial^2}{\partial \alpha_s^2} \beta(\alpha_s), \tag{4.17}$$

where (4.16) is in agreement with [3]. For γ_0 we cannot give a closed formula but a three-loop result. In the context of previous calculations [1, 2] we computed the contact term of C_0^{GG}, which equals Z_0, up to three-loop level:[9]

$$\begin{aligned}
Z_0 =& \frac{n_g}{16\pi^2} \left\{ \frac{1}{4\varepsilon} + \frac{a_s}{\varepsilon} \left(\frac{17 C_A}{32} - \frac{5 n_f T_F}{24} \right) \right. \\
&+ \frac{a_s^2}{\varepsilon} \left(\frac{11}{96} C_A^2 \zeta_3 + \frac{22351 C_A^2}{20736} - \frac{7}{24} C_A n_f T_F \zeta_3 - \frac{799 C_A n_f T_F}{1296} \right. \\
&\left. + \frac{1}{4} C_F n_f T_F \zeta_3 - \frac{107}{288} C_F n_f T_F + \frac{49 n_f^2 T_F^2}{1296} \right) \\
&+ \frac{a_s}{\varepsilon^2} \left(\frac{n_f T_F}{12} - \frac{11 C_A}{48} \right) \\
&+ \frac{{a_s}^2}{\varepsilon^2} \left(-\frac{833 C_A^2}{1152} + \frac{73}{144} C_A n_f T_F + \frac{1}{12} C_F n_f T_F - \frac{5}{72} n_f^2 T_F^2 \right) \\
&\left. + \frac{{a_s}^2}{\varepsilon^3} \left(\frac{121 C_A^2}{576} - \frac{11}{72} C_A n_f T_F + \frac{1}{36} n_f^2 T_F^2 \right) \right\},
\end{aligned} \tag{4.18}$$

[9] Only the Adler function of C_0^{GG} with the gauge group factors set to their QCD values was presented explicitly in [1].

the full Wilson coefficient being

$$
\begin{aligned}
C_0^{\mathrm{GG}} = Z_0 + \frac{n_g}{16\pi^2}\Bigg\{ & \frac{l_{\mu q}}{4} + \frac{1}{4} \\
& + a_s\left(\frac{11}{48}C_A l_{\mu q}^2 + \frac{73 C_A l_{\mu q}}{48} - \frac{3C_A\zeta_3}{4} + \frac{485 C_A}{192}\right. \\
& \left. - \frac{1}{12}l_{\mu q}^2 n_f T_F - \frac{7}{12}l_{\mu q} n_f T_F - \frac{17 n_f T_F}{16}\right) \\
& + a_s^2\left(\frac{121}{576}C_A^2 l_{\mu q}^3 + \frac{313}{128}C_A^2 l_{\mu q}^2 - \frac{55}{32}C_A^2 l_{\mu q}\zeta_3 + \frac{37631 C_A^2 l_{\mu q}}{3456}\right. \\
& - \frac{2059}{288}C_A^2\zeta_3 + \frac{11}{64}C_A^2\zeta_4 + \frac{25}{16}C_A^2\zeta 5 + \frac{707201 C_A^2}{41472} \\
& - \frac{11}{72}C_A l_{\mu q}^3 n_f T_F - \frac{85}{48}C_A l_{\mu q}^2 n_f T_F - \frac{1}{8}C_A l_{\mu q} n_f T_F \zeta_3 \\
& - \frac{6665}{864}C_A l_{\mu q} n_f T_F + \frac{169}{144}C_A n_f T_F \zeta_3 - \frac{7}{16}C_A n_f T_F \zeta_4 \\
& - \frac{7847}{648}C_A n_f T_F - \frac{1}{8}C_F l_{\mu q}^2 n_f T_F + \frac{3}{4}C_F l_{\mu q} n_f T_F \zeta_3 \\
& - \frac{131}{96}C_F l_{\mu q} n_f T_F + \frac{41}{24}C_F n_f T_F \zeta_3 + \frac{3}{8}C_F n_f T_F \zeta_4 \\
& - \frac{5281 C_F n_f T_F}{1728} + \frac{1}{36}l_{\mu q}^3 n_f^2 T_F^2 + \frac{7}{24}l_{\mu q}^2 n_f^2 T_F^2 \\
& \left. + \frac{127}{108}l_{\mu q} n_f^2 T_F^2 + \frac{4715 n_f^2 T_F^2}{2592}\right)\Bigg\},
\end{aligned}
\tag{4.19}
$$

which is known from [18],[19] and [6] for the special case of the gauge group factors replaced by their QCD values. This leads to

$$
\begin{aligned}
\gamma_0 = {} & \frac{n_g}{4} + \frac{1}{48}a_s n_g(51 C_A - 20 n_f T_F) \\
& + \frac{a_s^2 n_g}{6912}\left(C_A^2(2376\zeta_3 + 22351) - 16 C_A n_f T_F(378\zeta_3 + 799)\right. \\
& \left. + 8 n_f T_F(648 C_F\zeta_3 - 963 C_F + 98 n_f T_F)\right).
\end{aligned}
\tag{4.20}
$$

5 Conclusions

I have presented a derivation of an additive counterterm $Z_{11}^{\mathrm{L}} O_1^{\mathrm{B}}$ needed to renormalize the correlator of two scalar gluonic operators O_1 using the path integral formalism and extending the ideas of [3]. This counterterm explains and absorbs the divergences found in the Wilson coefficient C_1^{GG} in [1, 2]. A simple closed formula expressing Z_{11}^{L} to all orders through the QCD β-function was presented as well. Finally, the anomalous dimensions γ_{11} and γ_{11}^{L} for the correlator of two operators O_1 were expressed through the QCD β-function to all orders and the anomalous dimension γ_0 was computed at three-loop order.

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